Mechanics of Vertical-moving Fluidized Systems

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Fluidization, more specifically the fluidization of beds of solid particles and catalysts, has become a subject of wide interest in chemical engineering during the past few years. Its major practical application has thus far been in the catalytic cracking of petroleum fractions but it is being studied and developed as a technique for conducting a wide variety of other reactions involving solids and fluids, especially solids and gases. Its broad ramifications and its basic character have not as yet been widely appreciated.

The fundamental importance of the phenomenon of fluidization to the majority of chemical engineering operations and processes and hence to phase-change separations in which two phases must be brought into contact is now recognized. Any mass of particles, whether they be solid, liquid, or gas, which are suspended in a continuous fluid because of the motion of the fluid between the particles, represents and can be classified as a fluidized system (7).

This conclusion means that a multitude of operations involving masses of particles, droplets, or bubbles suspended in a fluid can be classified as fluidized systems. Examples are a liquid-liquid extraction tower, the fluidized catalytic-cracking bed of finely divided solids, a sieve-plate tower, mixing of two liquids or solid particles with liquid in an agitated tank, solids settling through a liquid (sedimentation), gas bubbles rising through a liquid (aeration), pneumatic transport of solids, and spray drying.

All such systems are controlled by and subject to the same fundamental combination of forces and their characteristics and behavior are determined basically by the same general principles. This is so irrespective of the relative direction of motion of the particles and fluid with respect to each other and to the walls of the containing vessel. The requirement is that relative motion or slip exist between particles and fluid. It is theoretically possible to generalize the basic properties of such systems in terms of the geometry, the physical properties of the system, and the flow rates regardless of whether the particles are solid, liquid, or gaseous.

In the present paper the authors will define and develop in detail the basic relationships that characterize any ideal fluidized system, an ideal system being defined as one in which nonporous, rigid spheres are fluidized in a particulate manner by an incompressible fluid. This will serve as a model analogously to the perfect gas law or Raoult's law for solutions. This ideal case will, in general, be more closely approached for solid particles or very small liquid droplets where the density difference between particle and fluid is not too great.

With the relative direction of motion of particles and fluid with respect to each other and the containing walls as a basis, the possible types of vertical fluidized systems are formulated. The slip velocity is shown to be the fundamental variable in all types of systems.

Based upon the experimental determination of the fraction holdup (or void fraction) and the superficial fluid velocity for an unfed, bottom-restrained fluidized system, a graphical procedure is outlined for the preparation of a generalized operational diagram. This diagram can be used for predicting and comparing the operating mechanics and characteristics of all ideal vertical fluidized systems. Experimental data for the fluidization of small solid particles with water are presented as validation of the basic theories.

The fundamental concepts will not be altered by complications introduced by a departure from the spherical or distortion and expansion of liquid and gas bubbles in motion. While very important in many practical cases, they are secondary effects determined by particular interrelationships of the forces involved and do not change the basic concepts.

Since the development of the theory and the completion of the experimental work described in this paper, Mertes and Rhodes (4) have published a treatment of moving solids-fluid systems. Their work lends additional support to the theory and generalizations presented in this paper.

BASIC QUANTITIES IN FLUIDIZATION

The void fraction ϵ and its opposite, the holdup $(1 - \epsilon)$, within a given volume are important characteristic properties of any fluidized system. To date they must be measured experimentally and correlated with the pertinent physical and flow factors, calculations that correspond to evaluating an equation of state. The upper limit of ϵ is the fully expanded bed, or 100% voids, conversely a holdup of zero. The lower limit, as will be seen later, depends upon the type of fluidized system.

The total pressure drop in a fluidized system is the sum of wall friction and hydrostatic head. On the basis of neglecting the very small amount of wall friction, the pressure drop per foot of height is given by

$$\frac{\Delta P}{z} = (1 - \epsilon)(\rho_d - \rho_f) \qquad (1)$$

where

$\Delta P/z$ = pressure drop per unit height
ρ_d = density of discontinuous phase
ρ_f = density of continuous phase

If the subdivided phase is lighter than the continuous fluid, $\rho_d < \rho_f$, the pressure drop occurs in the opposite direction, i.e., from top to bottom of the bed. Thus the pressure drop and the holdup are linearly related.

Of further interest are the velocities of the two phases. V_f' may be called the *superficial fluid velocity* and V_d' the *net average superficial vertical-particle velocity.* Both these velocities are based upon the total cross section of the empty tube. The actual velocities are then given by

V_f = fluid velocity = V_f'/ϵ

V_d = average particle velocity $= V_d'/(1 - \epsilon)$

Also the vectoral difference between the fluid velocity and the solids velocity (with the convention of upward flow as positive) may be defined as the slip velocity,

$$V_s = \text{slip velocity} = V_f - V_d$$
$$= V_f'/\epsilon - V_d'/1-\epsilon \quad (2)$$

As can easily be shown, for spherical particles, the specific contact area varies with the fraction voids in the following way

$$\bar{a} = \frac{6(1-\epsilon)}{D_p} \quad (3)$$

where

$\bar{a}$ = specific contact area per unit volume
D_p = particle diameter

Since $\bar{a}$ varies linearly with the amount of holdup $(1-\epsilon)$, the holdup is a prime factor in determining the contact area for mass transfer or mass transfer efficiency in any particle-type contacting device. In addition $\bar{a}$ is directly proportional to $\Delta P/z$ in such equipment. The importance of examining the behavior of the holdup for various operating conditions is immediately apparent.

TYPES AND CLASSIFICATION OF VERTICAL-MOVING FLUIDIZED SYSTEMS

The previous description of an ideal vertical fluidized system leads to the recognition of certain basic types and classes of systems. Two standard types may be postulated: (1) free and (2) mechanically restrained. The relative direction of motion of particle and fluid relative to the walls then leads to several types under each category. With the convention of upward flow assumed as a positive quantity and downward as negative and $\rho_d > \rho_f$, these categories and the slip velocity for each may be classed as follows:

Free Systems

Free systems require no constraint in the flow path with the particle feed rate controlled externally at the particle inlet. The rate of particle addition must be determined by the net movement of the volume of particles through the fluidizing zone.

Countercurrent: $V_s = V_f - (-V_d)$. Particles and fluid are fed at opposite ends of the tower. The relative position of feed depends on the density values. If $\rho_d > \rho_f$, then particles are fed at the top of the tower and flow downward. Free settling is a special case of countercurrent flow in which the fluid velocity V_f is set equal to zero. This assumes that the liquid being displaced by the accumulation of particles is removed without being allowed to pass through the dynamic contacting zone.

Concurrent Cogravity: $V_s = -V_f - (-V_d)$. This represents a relatively new technique for contacting. If $\rho_d > \rho_f$, both particles and fluid are fed at the top of the tower and move downward together.

Concurrent Countergravity: $V_s = V_f - (V_d)$. This is generally referred to as *solids transport*. If $\rho_d > \rho_f$, both particles and fluid are fed at the bottom of the tower and move upward together.

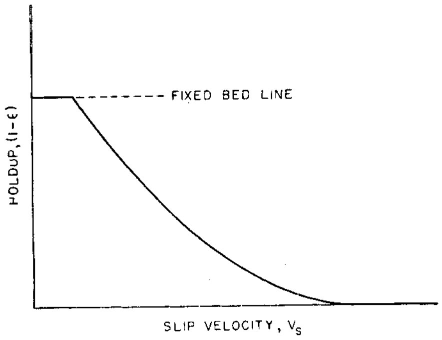

Fig. 1. Schematic representation of holdup $1-\epsilon$ vs. the slip velocity V_s.

Mechanically Restrained Systems

In these systems the flow path of the particles is mechanically restrained and the particle feed rate is externally controlled at the particle exit.

Unfed, Bottom Restraint: $V_s = V_f - (0)$. If $\rho_d > \rho_f$, the fluid is fed at the bottom of the tower and passes through a particle impermeable membrane or screen and then upward through a fixed quantity of particles. Even though there is motion of the particles within the bed, there is no net motion with regard to the walls. Thus the particle velocity V_d may be set equal to zero. This type of operation has received considerable experimental and theoretical study and represents in many people's minds the only form of fluidized system. If the restraining screen is removed, the operation degenerates into a free system, which could be maintained only by continuously feeding particles to the top of the tower.

This unfed system has one less degree of freedom than the previously described free systems. Whereas in countercurrent and concurrent flow both the fraction void and the particle velocity are variable, only fraction void can be changed in batch fluidization, as the net particle velocity is always zero. In terms of pressure drop, the nonfed bed always has a constant value of $\Delta P/z$. This is a result of the constant weight of the bed and the subsequent changes in z with V_f. For a continuously fed bed the length z is constant and the weight or holdup in the bed varies with V_d and V_f. Hence both $\Delta P/z$ and $1-\epsilon$ vary with V_d and V_f.

Bottom Restraint, Downward Particle Feed: $V_s = V_f - (-V_d)$. This corresponds to feeding liquid at the bottom of the tower (if $\rho_d > \rho_f$) and feeding particles at the top. In the steady state condition particles are withdrawn at the bottom at a controlled rate and fed at the top at the same rate to maintain a constant inventory of particles in the tower.

Bottom Restraint, Upward Particle Feed: $V_s = V_f - (V_d)$. For $\rho_d > \rho_f$, particles are continuously withdrawn at a controlled rate from the top of a bottom-restrained fluidized bed and constantly fed at the same rate at the bottom. This represents transport of the particles by the fluid.

Bottom Restraint, Concurrent, Cogravity Feed: $V_s = -V_f - (-V_d)$. Solid particles, $\rho_d > \rho_f$, are fed at the top of the bed with the fluid. Particles are withdrawn from the bottom at the same rate.

Top Restraint, Cogravity Particle Flow: $V_s = V_f - (0)$. For $\rho_d > \rho_f$, this case has significance only if V_f exceeds the transport velocity of an individual particle. Under these conditions a packed bed will be formed below the restraining screen. For conditions below the transport velocity the system will behave as if the screen were not present.

Top and Bottom Restraint, Fluid Upward: $V_s = V_f - (0)$. A variety of possible operations exists here: a fully packed bed between the two screens, a packed bed occupying only a part of the volume, or a fully fluidized system. The actual situation will depend on the total inventory of particles between the screens as compared with the available volume.

Slip Velocity as the Characterizing Parameter

The relationship between the operating mechanics of all types of systems is determined by the same basic laws. This can be illustrated by considering the behavior of a single particle suspended in a fluid. This is analogous to consideration of a very dilute solution or a high vacuum for a gas and can serve as a reference condition.

For a single spherical particle falling through a quiescent fluid, nonmoving with respect to the walls, an acceleration period is followed by a regime in which the particle moves at a constant velocity, which is the terminal, or free-fall, velocity V_t. By equating the vertical gravitational force downward to the vertical drag upward, this velocity may be calculated; i.e., for viscous flow

$$V_t = \frac{(\rho_d - \rho_f) g D_P^2}{18\mu_f} \quad \text{(Stokes's Law)}$$

As V_f, the fluid velocity, is zero, this equation predicts the particle velocity relative to the walls.

If the fluid is set in motion countercurrently upward relative to the walls, the velocity of the particle with respect to the walls V_d is reduced and when $V_f = V_t$ the particle is stationary, or $V_d = 0$. This fluid velocity is known as the *transport velocity* and is identical with the free-fall velocity V_t. This may be expressed as

$$V_d = V_t \quad \text{for} \quad V_f = 0 \quad \text{(free-fall)}$$

and

$$V_f = V_t \quad \text{for} \quad V_d = 0 \quad \text{(transport)}$$

At any intermediate value of V_f between zero and the transport velocity or, in fact, at any fluid velocity with respect to the walls, the relative or slip velocity of fluid past the particle is constant and equal to the free-fall, or transport, velocity. The slip velocity, as defined by Equation (2), is thus

$$V_s = V_f - V_d = \text{constant}$$

Thus the slip velocity, or the relative velocities of particle and fluid to each other, is the invariant parameter in determining the behavior and properties of the system. The particle knows the movement of only the fluid and not the walls and does not know whether it is moving relative to the latter or not.

For a mass of particles suspended in a fluid stream one may postulate the presence of a third group of forces, namely, the forces exerted by the particles on one another. This would cause an additional vertical component opposite to the fluid motion. In the present work it is proposed that even in a multiparticle fluidized system the invariant parameter of operation is still the slip velocity as defined by Equation (2). In other words, any one of the forms of a fluidized system described above will at the same value of the slip velocity have the same void fraction ϵ or the same holdup $(1 - \epsilon)$ for a given fluid-particle system.

In mathematical terminology this dependence of holdup on the slip velocity, no matter what the type of operation, can be expressed as

$$(1 - \epsilon) = \phi(V_s) \tag{4}$$

where $\phi(V_s)$ represents a functionality of V_s. Graphically Equation (4) can be schematically represented by Figure 1. The upper limit of holdup on this curve is the fully packed bed and the lower limit is an empty bed.

Equations (2) and (4) can be combined and plotted graphically. The resulting diagram defines on a single plot the relation between holdup and flow rates existent over any flow regime for all types of vertical free or restrained systems. Furthermore, this diagram defines the limits of operation of each type of system and the range of fluid velocities over which each can exist. Any fluid-particle system will exhibit this characteristic diagram, which can be quantitatively plotted to scale if the relation expressed by Equation (4) is known.

PREPARATION OF GENERALIZED OPERATIONAL DIAGRAM

While there are a number of methods available for preparing a generalized operational diagram, the technique to be illustrated here is the most convenient and rapid. Equation (2)

$$V_s = \frac{V_f'}{\epsilon} - \frac{V_d'}{1 - \epsilon} \tag{2}$$

may be rearranged to yield

$$V_d' = \frac{1 - \epsilon}{\epsilon} V_f' - (1 - \epsilon) V_s \tag{5}$$

For any single value of the holdup there will be only one value of the slip velocity V_s; therefore, for this value of the holdup Equation (5) represents a straight line, the slope of which will be $(1 - \epsilon)/\epsilon$ and the intercept on the V_f' axis ϵV_s.

In addition data are required for the system under consideration as represented by Equation (4). These could be obtained by experimentally determining $(1 - \epsilon)$ vs. the fluid velocity for a batch fluidization or a bottom-restrained, unfed bed. For this unfed operation $V_d' = 0$ and one may write

$$\left.\frac{V_f'}{\epsilon}\right|_{batch} = V_s = \phi^{-1}(1 - \epsilon)$$

Substituting into Equation (5) results in

$$V_d' = \frac{1 - \epsilon}{\epsilon} V_f' - (1 - \epsilon) \left.\frac{V_f'}{\epsilon}\right|_{batch} \tag{6}$$

For any value of $(1 - \epsilon)$ chosen, $(1 - \epsilon)/\epsilon$ can be calculated. With the value of $V_f'{}_{batch}$ corresponding to the chosen $(1 - \epsilon)$, the straight line as represented by Equation (6) is immediately defined. This procedure can be continued for all values of $(1 - \epsilon)$ from zero to the value for the fully packed bed. The result is a series of lines each at a constant value of the holdup. Figure 2 shows such a diagram for $\rho_d > \rho_f$. Each quadrant in this diagram is labeled as to the possible types of operation. The intercept described by ϵV_s is for $V_d' = 0$.

It is to be realized that Figure 2 holds only for one system, i.e., one particle size fluidized by a single fluid. There is partial experimental evidence that if the plot is made with V_f'/V_t vs. V_d'/V_t as coordinates then this chart becomes a general one for all types of systems although predicted on the basis of experimental data for only one system.

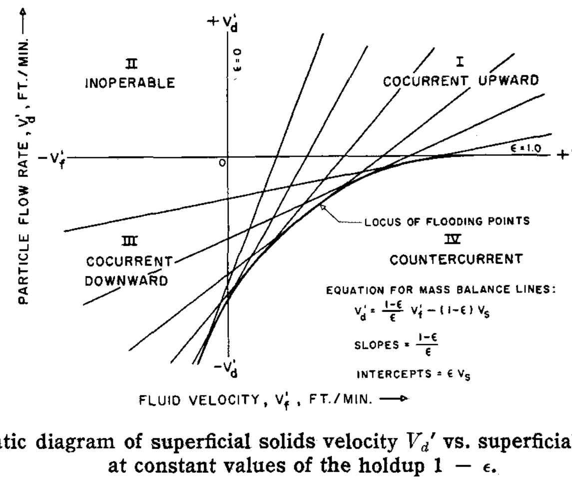

Fig. 2. Schematic diagram of superficial solids velocity V_d' vs. superficial fluid velocity V_f' at constant values of the holdup $1 - \epsilon$.

THE GENERALIZED HOLDUP—FLOW-VELOCITY CURVE

By use of Figure 2 a cross plot has been constructed in which the fraction holdup is plotted as ordinate vs. the superficial fluid velocity as abscissa with superficial solids velocity as the parameter. This diagram is represented in Figure 3 and is titled the generalized operational diagram for vertical moving fluidized systems. It shows the relationship among and the limits of all types of vertical fluidized systems.

The ordinate of holdup $1 - \epsilon$ has values from zero to the figure corresponding to a fully packed bed. The abscissa of superficial fluid velocity V_f' has both negative and positive values. The positive values may be classified as smaller or greater than the transport velocity of a single particle. Shown above the graph are schematic representations of the different regimes or zones of fluidization. All free-system operations are bounded by the area $\beta abdz\beta$ and the area to the right of jqd. Any other area on the diagram represents operation with mechanical restraint of some type.

Before a discussion of the various regimes in this generalized diagram it may be best to note the significance of certain of the areas, points, and lines.

1. For negative values of V_f' the fluid moves downward and for positive values upward.
2. At $V_f' = V_t =$ point d, particles and fluid both move upward.
3. To the left of $V_f' = 0$ particles and fluid both move downward.
4. Point c is the commonly determined fluidizing velocity of an unfed supported

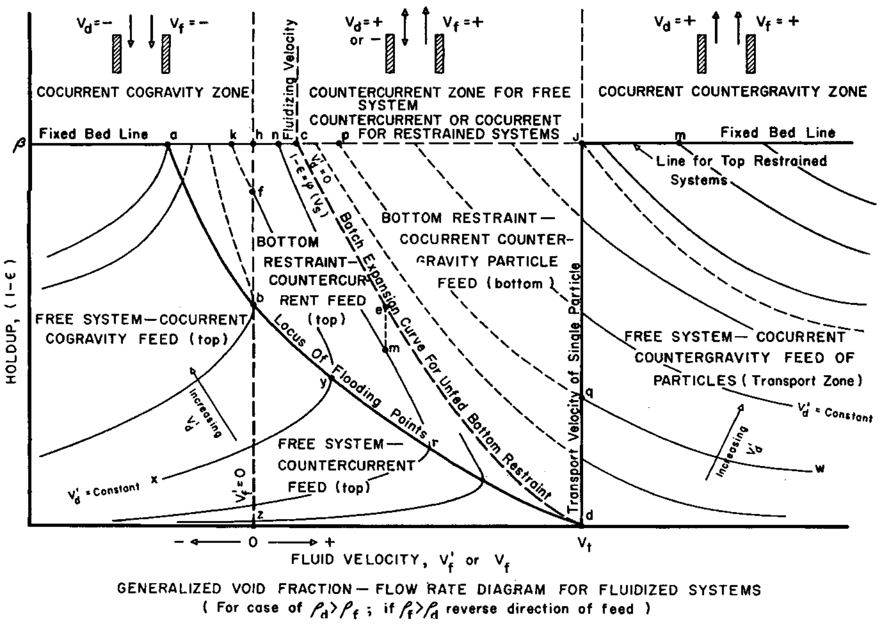

Fig. 3. Schematic generalized operational diagram for vertical-moving fluidized systems.

bed and the line *cd* represents the batch expansion curve.

5. No free system can exist above curve *abd*.

6. Curves such as *xy* or *wqp* show the variation in holdup for various combinations of fluid velocity, in each case at a constant particle velocity.

It is important to note that fluidization is a single continuous phenomenon extending from the fixed bed on one side through a region of increasing fluid velocity to a fixed bed at the other limit. Each of the different types of vertical flow systems represents a different regime within limits defined by the basic laws. Thus countercurrent unrestricted fluidization, e.g., the spray tower, has concurrent countergravity and cogravity fluidization as its limits. A mechanically supported bed has the fixed bed and transport as its limits.

The free concurrent-cogravity and free countercurrent systems are represented by the areas $\beta abz\beta$ and *bydzb*, respectively. In each area the lines of constant superficial solids rate increase in numerical value from bottom to top and from right to left. Thus a vertical line in either regime shows the change of holdup with changing particle feed rate at a constant fluid rate. Such towers are operable only in the region to the left of curve *abd*, which is termed the locus of the flooding points. Thus, point *y* represents the flooding point for the solids feed rate indicated by line *xy*. The flooding point is defined as the maximum fluid rate possible at a given solids rate in a free fluidized system. The significance of this flooding line will be discussed later. A further increase in V_f' causes $1 - \epsilon$ to decrease along *yd* and the particle throughput no longer equals the rate of particle feed since a portion is rejected by the system. The rejection rate increases with increasing V_f' until it equals the rate of feed at V_t, which is equal to the free-fall velocity of a single particle. Point *b* represents the maximum holdup and particle feed possible in a stationary column of fluid. Note that the concurrent cogravity holdups may be much larger than the countercurrent-system holdups. This indicates a possible advantage for this type of contacting.

The area to the right of the line *jqd* represents the regime of concurrent-countergravity flow, commonly referred to as the *transport region.* Lines of constant superficial solids rate, such as *wq*, increase in numerical value from bottom to top and from left to right. In this case the holdup decreases as the fluid velocity increases for a fixed rate of particle feed. It is possible to have any combination of holdup from that of a packed bed to zero depending upon the particular combination of flow rates. A flooding point corresponding to that for countercurrent operation apparently does not exist.

Bottom-restraint–countercurrent operation is represented by the region bounded by *hcdbh*. This type of system can be achieved by starting with a nonfed fluidized bed and then adding solids at the top and withdrawing from the bottom. Initially a condition on the batch fluidization curve such as point *e* will be established. As the solids are fed the bed adjusts vertically downward to a new condition, say point *m*. For this particular solids rate, variations in the fluid velocity will cause the holdup to move either along *mn* or along *mr*. Point *r* represents a limiting point in both the countercurrent and the bottom-restrained–countercurrent feed systems. At the same fluid velocity and the same particle velocity the bottom-restrained-countercurrent system gives higher holdups than the free system. Neither, however, give holdups as large as the nonfed–bottom-restrained condition at the same fluid velocity.

Bottom-restraint–concurrent-countergravity operation is represented by the region bounded by *cpjqdc*. By introduction of particles to the bottom of a batch-fluidized bed and withdrawing from the top, a bottom-restrained transport condition is achieved. The holdups in this case will be larger than for the batch-fluidized bed. Higher holdups at a lower fluid velocity than in the free system are also possible. The limits of operation of superficial fluid velocity are the fluidizing velocity, point *c*, and the transport velocity, point *d*. Above the transport velocity the presence or absence of a mechanical restraint will not influence the stability of the system.

Bottom restraint, concurrent-cogravity feed is represented by the region *ahba*. This system can be achieved by starting with a fixed bed sitting on a bottom restraint. Particles are then withdrawn from the bottom and added at the same rate. This corresponds to moving from point *h* to say point *f*, a fluidized bed. The particle rate must, of course, be greater than the rate at *h*. Adding fluid to the top of the bed will move the holdup along the line *fk*. The maximum fluid velocity possible is given by point *a*.

Consideration may be given to the case of top restraint or top and bottom restraint, but the practical implications of these types of operations are not apparent and so these operations will not be discussed.

Sedimentation can also be represented on such a chart. The constant rate-of-sedimentation curve would lie to the right of the zero velocity point, as the fluid is set in motion countercurrent to the walls by displacement as particles settle through it.

Any system in which the particles are fed at either terminal is restricted in operation to the flow conditions below the limiting void of the flooding point. If, however, the particles are fed at an intermediate point between the terminals, a restriction is removed or a degree of freedom added. This permits a tower of this design to operate either as a countercurrent or a concurrent fluidized bed. It may be transferred from one system to the other by an adjustment of either the fluid or particle flow. Wilhelm and Valentine (*9*) have studied a system of this type and have shown that it may be transferred from one or the other system at will. They also showed that the void-fraction–fluid-velocity relationship closely parallels that of screen-supported fluidization and passes through a minimum with gas flow for any solids flow in the region where transition from one type of fluidization to the other occurs.

The reader's attention is called to the recent work of Flinn (*1*), who has summarized in detail many of the aspects of the operational diagram and the modes of operation discussed in this paper.

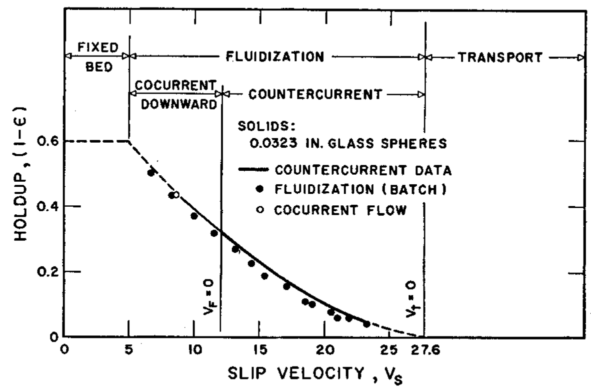

Fig. 4. Data of Price (*5*), holdup $(1 - \epsilon)$ vs. slip velocity V_s.

CONCEPT OF "MINIMUM VOID"—LIMITING FLOW OR FLOODING IN FREE COUNTERCURRENT SYSTEMS

The void fraction in a mechanically supported bed can vary from that of the fully packed bed (approximately 40% voids for most spherical solids) to the fully expanded condition of 100% voids. In contrast, in any continuously moving fluidized system there exists, for any particular size and combination of densities and fluid viscosities, a "minimum void" fraction (*5*) which the system cannot exceed. This minimum void fraction varies slightly with the flow ratio of particles and fluid V_d'/V_f' and density difference $\Delta\rho$ and usually lies in the region from 100% to approximately 80 to 85% voids. In none of the countercurrent systems thus far observed in these studies have void fractions less than about 70 to 75% been obtained; in most they correspond to 90 to 95% voids. This minimum void corresponds to a definite minimum possible slip velocity and represents the limiting flow which can be obtained through such a system. In other words this represents the flooding point or flooding velocity. Mathematically, this point can be described by

$$\left.\frac{d(V_f')}{d(1-\epsilon)}\right|_{V_d'} = 0 \qquad (7)$$

as can be seen in Figure 3. The locus of these points is represented by curve *abd* in Figure 3.

When the flow rates to the system exceed those corresponding to this minimum void, a portion of the particles fed is rejected by the system; this portion is refused by a tower of constant cross section and overflows the tower at the point of feed. If the feed terminal of the tower is surmounted by a section of expanding cross section (a funnel), thus reducing the fluid velocity at this point, the rejected portion of the feed forms a fluidized bed in the funnel supported by the fluid bed below. The height above the tower proper varies with the rate of feed above the flooding point. With high-density lead particles and proper funnel dimensions it is possible to form a fixed bed in the funnel above the flooding point. When this occurs, the feed rate to the system is reduced to that corresponding to the flow of particles through a fixed orifice and control of the fluidization is lost. Flooding will also be a phenomenon of free concurrent cogravity systems (Figure 3).

The so-called "rejection" or "carry-back" point employed by some authors has no fundamental significance and is peculiar to some particular combination of design and operating conditions of a certain tower. The well-known minimum escaping surface or area necessary for a boiling or gas-aerated system is directly related to the minimum void and the heterogeneity of fluidization. Likewise in the dispersion of an immiscible liquid or solid particles in a batch-agitated vessel there is a minimum void which is an exemplification of the same basic phenomena.

The characterization and general correlation of the flooding point for a fluidized system to permit prediction is of utmost importance. This can be accomplished by use of Figure 2, which represents a family of constant-void-fraction lines intersecting in the countercurrent and concurrent-cogravity regimes. Since the flooding point is defined as the maximum throughput of solids at a given liquid throughput, the flooding point will be located on the tangent to the family of constant-void-fraction lines. This tangent curve is called the *locus of flooding points* on Figure 2 and can be located by drawing a smooth curve tangent in turn to each constant-void-fraction line. Thus the entire flooding curve can be obtained from data for an unfed bottom-restrained bed.

An analytical expression for the flooding curve could be obtained if the equation-of-state curve, Equation (4), is known. By using Equations (4) and (5), and carrying out the differentiation indicated as the necessary condition at flooding, Equation (7), one can evolve an algebraic expression.

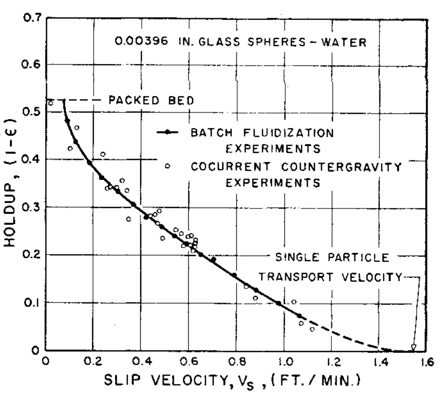

Fig. 5. Data of Struve (*6*), holdup $(1 - \epsilon)$ vs. slip velocity V_s.

GENERAL CONCLUSIONS REGARDING FLUIDIZED SYSTEMS

1. Free countercurrent systems are generally restricted to a void range between ϵ = 100% and approximately ϵ = 70 to 80%.

2. Such systems can be formed only from the fully expanded or 100% void state and never from the fully packed state.

3. Similar restrictions exist at zero fluid feed rate, i.e., in a nonflowing column of fluid. Hence a bed of particles can never be passed through an unrestricted column of fluid in the fully packed condition. Only by introduction of a restriction in the column by a reduced opening or by concurrent-cogravity flow of the continuous fluid can a fully packed bed be formed. For a liquid-liquid system this would result in coalescnce. This opening and not the fluid dynamic conditions or the particle feed rate would then control the discharge rate independently of the bed height.

4. A mechanically supported bed may have any void fraction between the fully packed and 100%.

5. Concurrent-cogravity systems may attain a holdup equal to the fully packed condition; i.e., passing fluid downward may fully pack the bed, and increase the holdup and hence the contact area.

6. It will be noted that these theoretical curves predict that there will not be a single invariant fluidizing velocity. Instead this velocity will be determined by

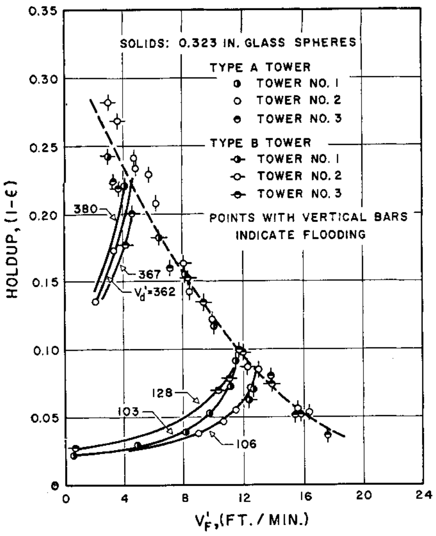

Fig. 6. Data of Price (5), holdup $(1 - \epsilon)$ vs. superficial fluid velocity V_f'.

countercurrent data, the solid points represent batch-fluidization data, and the single unfilled point is concurrent flow downward. The points of zero fluid velocity ($V_f' = 0$) and zero particle velocity V_t are indicated. In the concurrent downward bed the holdup exceeds either of the other systems and may approach that of a fully packed bed. This is an important point for it means that much larger contact areas for mass transfer are possible under these conditions.

Figure 5 represents data of Struve (6) once again plotted as holdup vs. the slip velocity. Raw-data points are shown for concurrent-countergravity flow for both free and bottom-restrained systems. The holdups fall around the smooth continuous curve representing batch-fluidization operation. Least-square treatment of the data gives a series of points which fall almost exactly on the continuous curve.

The behavior of these same experimental data as predicted by the generalized operating chart, Figure 3, is also of interest.

being studied in terms of the theoretical predictions of the generalized operational diagram.

DESIGN CONSIDERATIONS

It is important to emphasize one phase of the results of the present theory. For known particle diameter, densities and viscosities, the holdup and hence ΔP may be predicted for any vertical-flow fluidized system from an experimental determination of holdup vs. fluid velocity for a mechanically supported unfed bed of particles and fluidizing fluid. Alternatively, any of the generalized correlations of holdup vs. slip velocity in the literature (2, 3, 8) may be used with due regard for the accuracy and region of fluidization for the particular measurements involved.

Either technique allows the complete determination of the generalized operational diagram from the data for the single type of fluidized system. Thus the design of a multitude of different types of moving fluidized systems may be easily achieved.

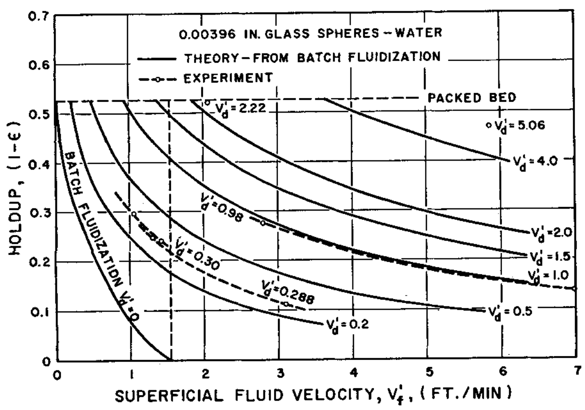

Fig. 7. Data of Struve (6), holdup $(1 - \epsilon)$ vs. superficial fluid velocity V_f'.

the flow rates and the mechanical arrangement of the system.

EXPERIMENTAL DATA

Until this point the present paper has dealt largely with the theoretical development of the behavior of ideal fluidized systems. Now some of the experimental data which have been collected in these laboratories will be examined to verify or disprove the validity of the theory. Only a brief resume of the experimental work will be presented, as detailed accounts will be the subject of further papers.

The systems under study were spherical glass beads fluidized by water. The ratio of the solid-particle diameter to the fluidizing-tower diameter was always small, i.e., at least 1/50, in order to minimize wall effects. These conditions represent a reasonable approach to the ideal system previously defined.

Figure 4 represents data of Price (5) with fraction holdup $(1 - \epsilon)$ plotted vs. the slip velocity V_s. The holdups fall on a continuous curve from $(1 - \epsilon)$ equal to approximately 0 to 60%. (This corresponds to void fractions of 99+ to 40%.) The solid curve represents a full range of Figure 6 represents the data of Price in the free countercurrent region. The dashed line shown is the best fit to the flooding data and very nearly coincides with the curve predicted from batch-expansion data. Figure 7 is a plot of the data of Struve in the transport region. In general, the agreement between predicted and experiment is good.

Thus the validity of a single invariant relationship as expressed by the equation

$$(1 - \epsilon) = \phi(V_s) \qquad (4)$$

would seem to be substantiated for the systems considered. Since only a single experimental point has been collected for the region of concurrent-cogravity flow a detailed investigation of this type of operation is in progress. The possible advantages for the cogravity system such as increased contact area and mass transfer and hence the possibility of achieving higher efficiencies in phase-contacting devices makes the elucidation of this region of the utmost importance.

In addition departures from the ideal representation are presently under investigation. The behavior of liquid particles passing through a second liquid with an accompanying deformation of the drops is

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